

The ISCO for Single Black Holes

$$\begin{aligned} \text{SBH: } E(x) &= \frac{1-2x}{(1-3x)^{1/2}} - 1 \\ &= -\frac{1}{2}x + \frac{3}{8}x^2 + \frac{27}{16}x^3 + \frac{675}{128}x^4 + \frac{3969}{256}x^5 + \dots \end{aligned}$$

$$E(x) \equiv \frac{H(x) - mc^2}{mc^2}, \quad x = \left(\frac{GM}{c^3} \Omega \right)^{2/3}$$

$$\text{ISCO: } \frac{dE}{dx} = 0 : \quad x_{\text{ISCO}} = \frac{1}{6}$$