Effects of spinning objects

$$\mathbf{P}_1\delta(\mathbf{x}-\mathbf{x}_1) \to (\mathbf{P}_1 + \tfrac{1}{2}\mathbf{S}_1 \times \boldsymbol{\nabla}_1)\delta(\mathbf{x}-\mathbf{x}_1)$$

$$H_{\mathrm{SO}} = \frac{2G}{c^2 R^3}(\mathbf{S}\cdot\mathbf{L}) + \frac{3GM_1M_2}{2c^2R^3}(\hat{\mathbf{S}}\cdot\mathbf{L})$$

$$\mathbf{S} \equiv \mathbf{S}_1 + \mathbf{S}_2, \;\; \hat{\mathbf{S}} \equiv \frac{\mathbf{S}_1}{M_1^2} + \frac{\mathbf{S}_2}{M_2^2}, \;\; \mathbf{L} \equiv \mathbf{R}\times\mathbf{P}, \;\; \mathbf{P} \equiv \mathbf{P}_1 = -\mathbf{P}_2$$

$$H_{\mathrm{SS}} = \frac{G}{c^2R^3}\left(\frac{3(\mathbf{S}_1\cdot\mathbf{R})(\mathbf{S}_2\cdot\mathbf{R})}{R^2} - (\mathbf{S}_1\cdot\mathbf{S}_2)\right)$$

$$H_{\mathrm{SS}}^{\mathrm{Kerr}} = \frac{GM_1M_2}{2c^2R^3}\left(\frac{3(\tilde{\mathbf{S}}\cdot\mathbf{R})(\tilde{\mathbf{S}}\cdot\mathbf{R})}{R^2} - (\tilde{\mathbf{S}}\cdot\tilde{\mathbf{S}})\right), \qquad \tilde{\mathbf{S}} \equiv \frac{\mathbf{S}_1}{M_1} + \frac{\mathbf{S}_2}{M_2}$$